Dual-Paraboloid Shadow Mapping

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Introduction and Background

• An improved perspective shadow mapping technique
• Less memory and render passes required
• Paper was written in 2002
Shadow Mapping

• Very fast to compute
• Acceptable quality
• Requires multiple passes for hemispherical and omnidirectional light sources
Simple Shadow Mapping

- Most basic form of shadow mapping
- Draw scene from light’s perspective
- Store depth buffer
- Compare when drawing shadows
Mapping for hemispherical and omnidirectional lights

- Spherical mapping
  - Single map, but heavy distortion
- Blinn/Newell Mapping
  - Different parameterization, but expensive
- Cube Mapping
  - Six maps, very expensive
- Dual-Paraboloid Mapping
  - Two maps, little distortion
Dual-Paraboloid Shadow Mapping

• Similar to traditional shadow mapping
• Needs only one pass per hemisphere
Dual-Paraboloid Shadow Mapping

- Analogy: the image obtained by an orthographic camera viewing a perfectly reflecting paraboloid.
Dual-Paraboloid Shadow Mapping

\[ f(x, y) = \frac{1}{2} - \frac{1}{2}(x^2 + y^2) \]
Dual-Paraboloid Shadow Mapping

Generating the Shadow Map.

- From 3D to texture coordinates:
  - we need the point \( P \) on the paraboloid that reflects a given direction \( \mathbf{v} \) towards the direction \( \mathbf{d} \).
Dual-Paraboloid Shadow Mapping

Generating the Shadow Map.

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For convenience we use the inverted direction.
Dual-Paraboloid Shadow Mapping

*Generating the Shadow Map.*

- From 3D to texture coordinates:
  - we need the point \( P \) on the paraboloid that reflects a given direction \( \mathbf{v} \) towards the direction \( \mathbf{d} \).

For convenience we use the inverted direction.

And transform everything to light coordinate space.

![Diagram of dual-paraboloid shadow mapping](image)
Dual-Paraboloid Shadow Mapping

Generating the Shadow Map.

Using the formula for the paraboloid:

Normal vector at \( P(x, y, z) \):
\[
\vec{n} = \frac{1}{z} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}
\]

Halfway vector at \( P(x, y, z) \):
\[
\vec{h} = \vec{d}_0 + \vec{v} = k \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \quad v_z \geq 0
\]
Dual-Paraboloid Shadow Mapping

Generating the Shadow Map.

• Halfway vectors for two connected paraboloids

\[ \mathbf{h} = \mathbf{d}_0 + \mathbf{v} \quad \mathbf{v}_z \geq 0 \]
\[ \mathbf{h} = \mathbf{d}_1 + \mathbf{v} \quad \mathbf{v}_z < 0 \]

where \( \mathbf{v} \) is the direction from the light to the point in light coordinate space

and

\[ \mathbf{d}_0 = (0, 0, 1) \]
\[ \mathbf{d}_1 = (0, 0, -1) \]
Dual-Paraboloid Shadow Mapping

Generating the Shadow Map.

• To get texture coordinates from halfway vector:
  • Divide x and y by the z-component
  • Map from [-1, 1] to [0, 1]

• Store distance [0,1] using near and far distances
Dual-Paraboloid Shadow Mapping

*Shadow Test*

- Do the same calculations for shadow testing
- Sample correct paraboloid texture using $z$ comparison
- Compare point depth to stored value
What does it offer?

• Advantages:
  • Less artifacts due to parabolic map
  • One map covers one hemisphere
  • Easy to implement
  • Low computational cost
  • Easily combined with usual shadow mapping techniques

• Disadvantages:
  • Still reduced precision around perimeter
  • Fails for very large polygons because of rasterization
Improvements

• Further improvements have been made:
  • “Practical Implementation of Dual Paraboloid Shadow Maps” – B. Osman et al. (2006)
• Original implementation needs sufficient geometry in both shadow caster and shadow receiver
• Eliminates need for shadow receivers to be tesselated
Improvements

• Original DPSM needs tessellation because vertex shader performs linear interpolation

• Solution suggests sending world-coordinates
  • These interpolate correctly and can be transformed in the pixel shader afterwards
Improvements

• Variance Shadow Mapping (VSM) can be applied to DPSM to achieve soft shadows
• Regular method for VSM can be applied, no difference compared to applying VSM to SSM.
Questions